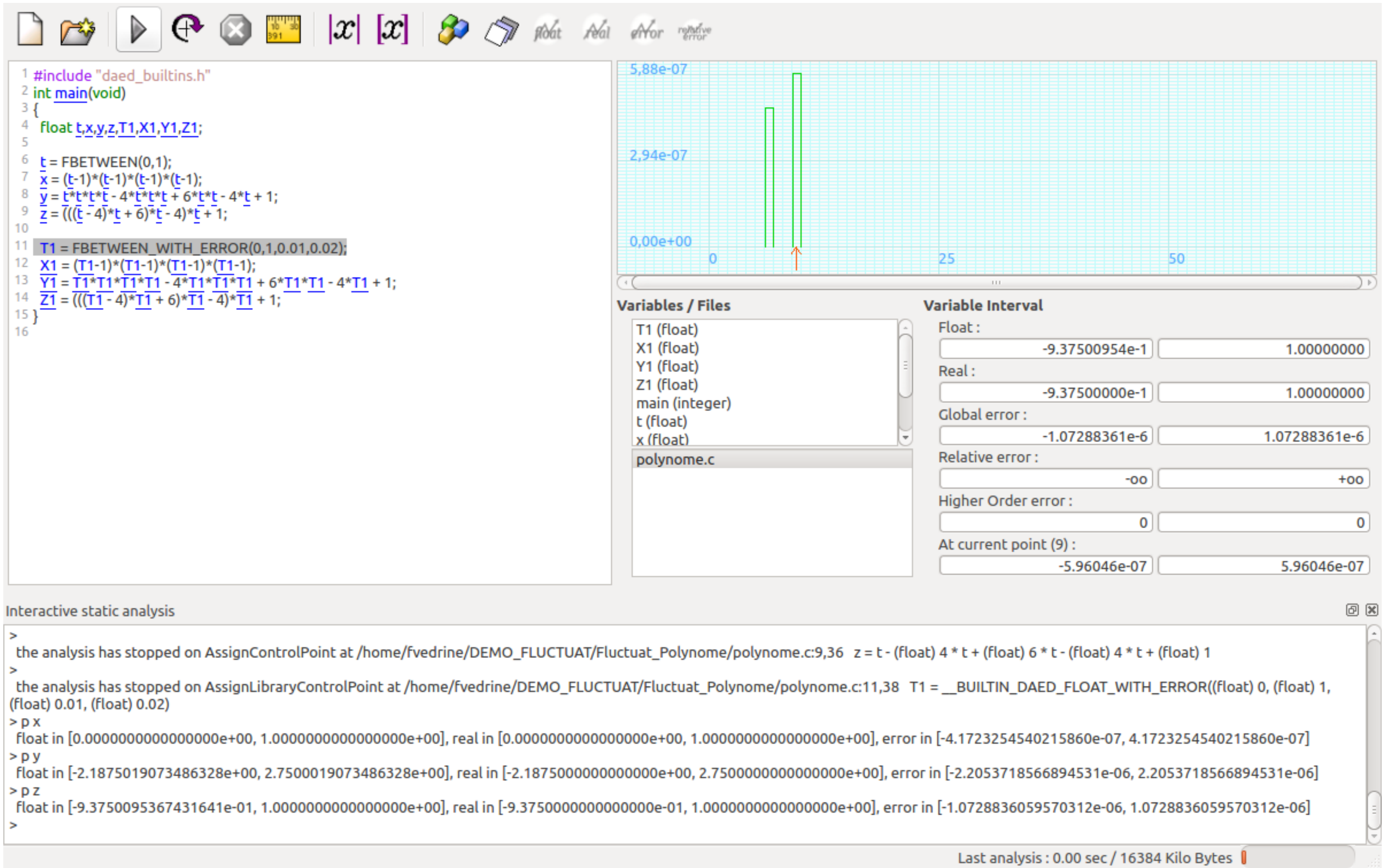
FLUCTUAT: VERIFICATION OF ACCURACY PROPERTIES OF NUMERICAL COMPONENTS AND SYNCHRONOUS EMBEDDED SOFTWARE

Journée précision numérique | Védrine Franck
Joint work with Eric Goubault & Sylvie Putot & Karim Tekkal + Maxime Jacquemin

VERIFICATION OF POLYNOMIAL APPROXIMATIONS

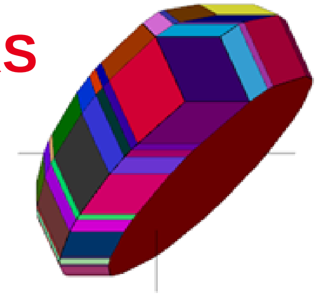


VERIFICATION OF INTERPOLATION TABLE

```
double y[10] = { 0.0,2.0,3.0,2.0,0.0,-2.0,-3.0,-1.9,0.1,2.1 };
double in = interval_with_error(0, 10, -1e-6, 1e-6), out;
SPLIT
int index = (int) in;
MERGE
if (index < 0) out = y[0];
else if (index >= 9) out = y[9];
else out = y[index] + (in - index) * (y[index+1] - y[index]);
MERGE
```

- **11 regular paths**
- **110 possible unstable branches**
- **20 effective unstable branches $\Rightarrow$ 40 unstable paths**
- **51 abstract interpretation of the code between** SPLIT/MERGE
- out: *float* $\in [-3.0, 3.0]$, *ideal* $\in [-3.0 - 1.11\times10^{-16}, 3.0]$,
  error $\in [-4.0\times10^{-6} - 4.85\times10^{-15}, 4.0\times10^{-6} + 4.44\times10^{-15}]$

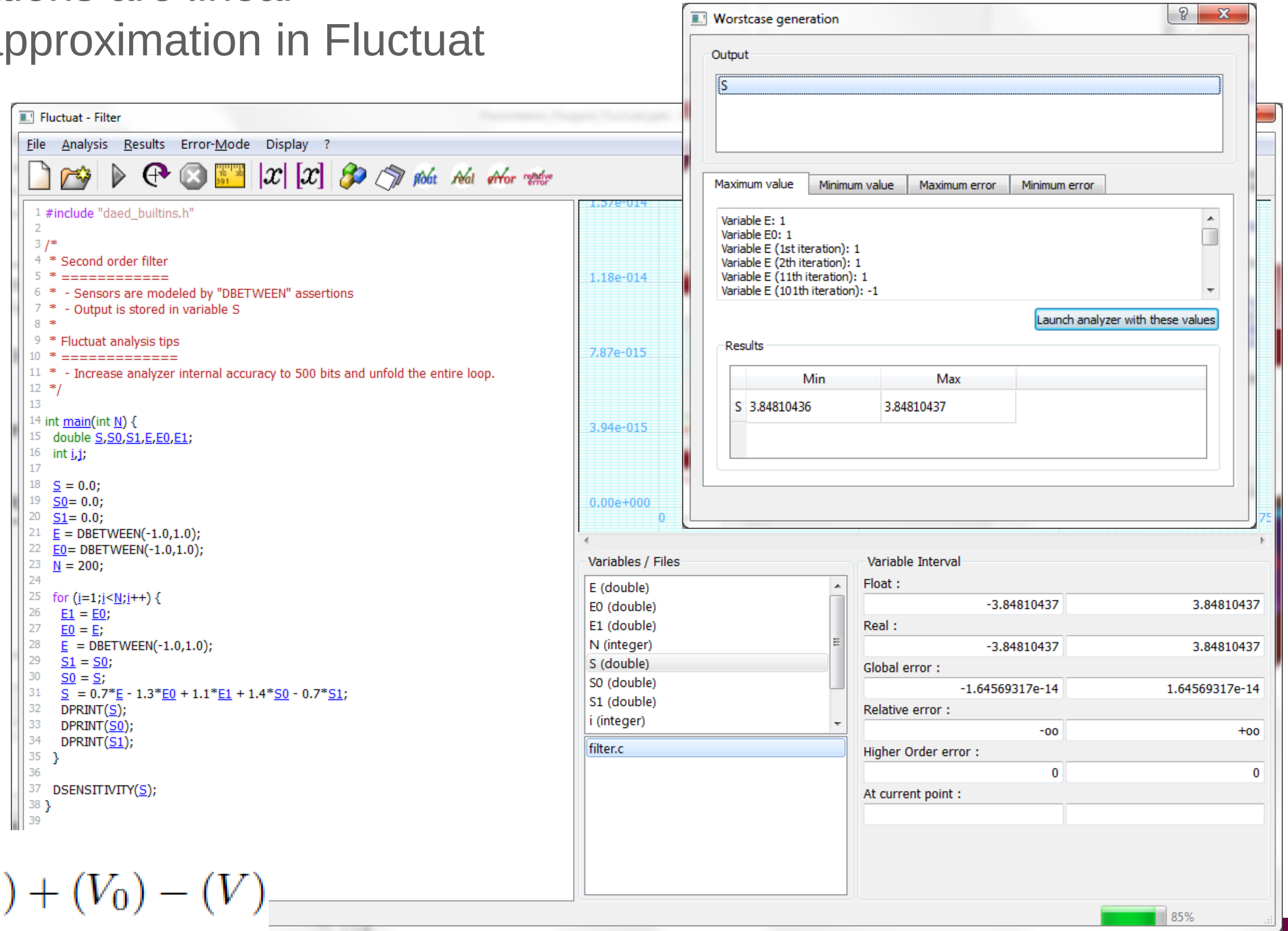
VERIFICATION OF CONVERGENT LINEAR FILTERS



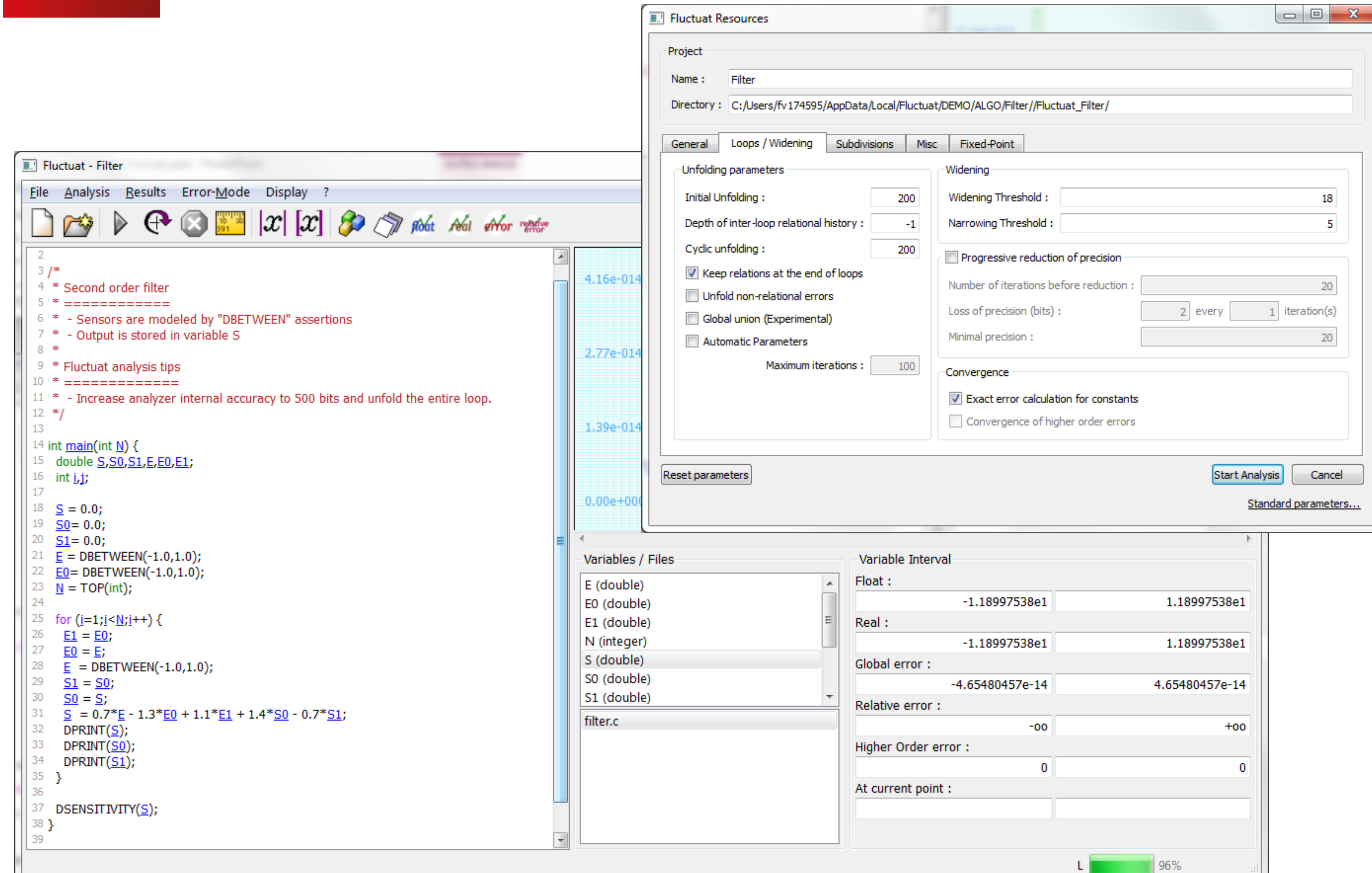
- **Why it works**
  - all computations are linear
  - almost no approximation in Fluctuat
  - domain and error = same type of transfer function
  - $1 + x + x^2 + \ldots + x^n$ bounded by $1/(1-x)$
- **Transfer function**

$$(M) = \begin{pmatrix} 0 & 1 \\ 1.4 & -0.7 \end{pmatrix}$$

$$S_n = \sum_{k=0}^{n} (M)^k.(V) + (V_0) - (V)$$

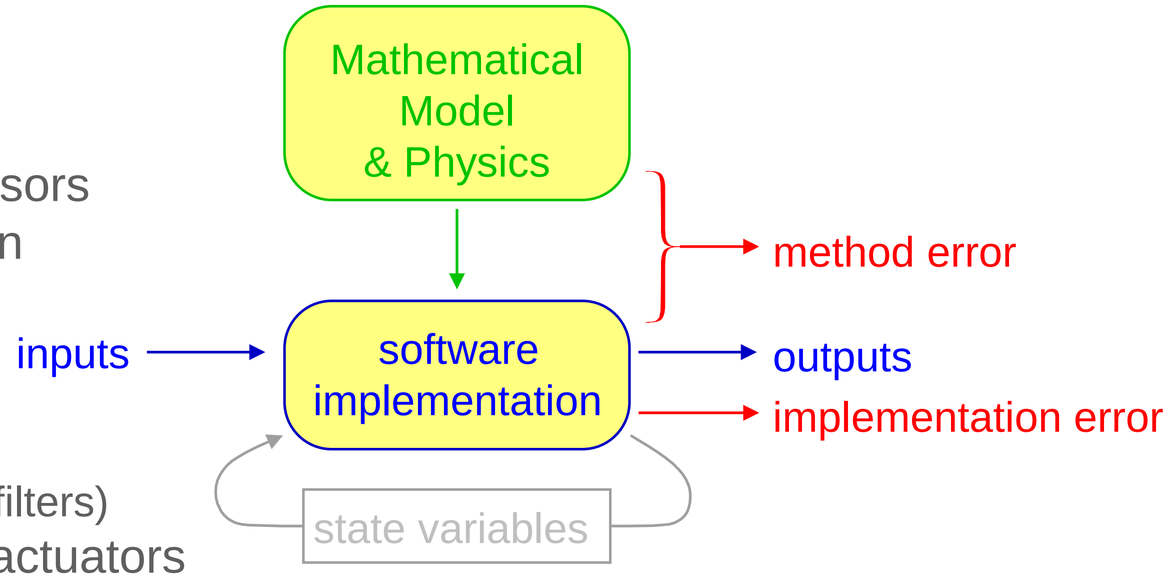


CONVERGENCE PROOF FOR CONVERGENT LINEAR FILTERS



EMBEDDED SYNCHRONOUS SYSTEM

- **Infinite loop**
  - Take inputs from sensors
  - Do some computation
    - in constrained time
    - without any allocation
    - read/write internal state variables (ex filters)
  - Produce outputs for actuators



- **Questions**
  - Is the accuracy of outputs bounded over time? within acceptable bounds?
  - Is the system robust ?
    - small perturbation of inputs produces a small perturbation of outputs

- **Objective**
  - Ensure that the implementation is conform to the model
  - Avoid to take irreversible decisions on wrong computations

FLUCTUAT CAN BOUND THE ACCURACY

- **if state variables only concern**
  - convergent linear filters of the inputs
  - and/or convergent linear filters of the outputs
- **if no discontinuous unstable branches can occur**
- **if all atomic operations respect the operation hypotheses**
  - no possible division by zero,
  - no negative square root
- **Then Fluctuat can bound the accuracy of the output**
  - for input values in full range
  - in theory for every simulation cycle,
    - unroll the loops on first cycles and see the evolution of error on the filters' state variables
    - depends on time, find adequate parameters (cyclic unfolding, widening and narrowing threshold) to achieve the proof for all cycles

GENERAL EMBEDDED SYNCHRONOUS SYSTEM

- **Fluctuat has many reasons to fail at bounding domains and accuracy for a full range of inputs**
  - acceptable bounds may not exist
  - accumulation of over-approximations

- **Apply the methodology from some test-cases**
  - compare with observed accuracy with MPFR
  - compare with stochastic intended accuracy with Verrou , Verificarlo, Cadna

  - guaranteed intended accuracy around the test-case
    - investigation and annotations for the unstable branches
    - verification of the results on the first iteration cycles

- **More and more complex system – ex geolocalization with solvers inside**

INTEREST OF USING SEVERAL TOOLS FOR VERIFICATION

- **To find bugs when different tools return unexpectedly different results**
  - ex: if you forget to std:: when calling the abs function,
    a discontinuous unstable branch will reveal an implicit int conversion

- **In a certification context (DO178C – avionics, FDA – medical, …)**

- **To progressively add annotations and go to proofs**

CONCLUSION

- **Fluctuat has shown that Abstract Interpretation can effectively deal with accuracy properties to infer and prove accuracy bounds**

- **Fluctuat has features that needs to be developed to target HPC codes**
  - ability to analyze parts of code on large input ranges
    - create a summary that computes an approximation of the precision loss of a function
    - challenge: more symbolic computations for the error
  - ability to keep a trace of the origins of the numerical errors (data flow)
    - to couple with debugging capabilities to explain abnormal behavior
    - challenge: avoid explosion of the number of noise symbols
  - ability to generate and simulate a worst-case
    - to couple with debugging capabilities
    - challenge: define a semantics different but close to IEEE-754 to run this worst-case
  - ability to generates dependencies from a specific parameter/input
    - challenge: adequate user-interface to quickly investigate these dependencies

Thanks for
your attention!

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Institut List | CEA SACLAY NANO-INNOV | BAT. 861 – PC142
91191 Gif-sur-Yvette Cedex - FRANCE
www-list.cea.fr