Dionysos.jl: a Modular Platform for Smart Symbolic Control

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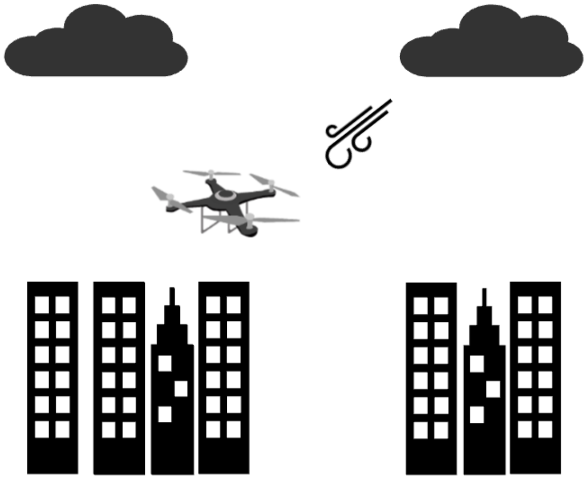
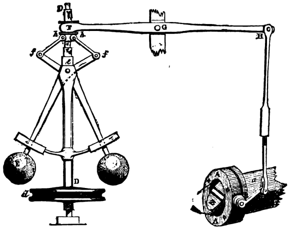
Control theory



The goal is to provide a generic procedure to design efficient controllers with formal guarantees.

A Paradigm Shift in Control Theory

Classical applications made the golden age of Control Theory



State space representation unleashed analytic approaches

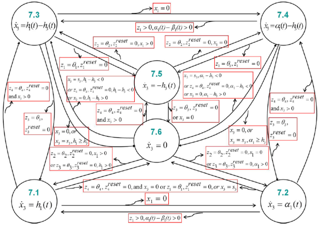
$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix} = \begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -5 & -26 & -5\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} + \begin{pmatrix}0\\ 0\\ 1\end{pmatrix} u$$

$$y = \begin{pmatrix}1 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$$

Cyber-Physical Systems paradigm shift

However modern applications are increasingly complex. . .

. . . and so are their models



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Learning to control (L2C)

We need a new control paradigm

Smart and Data-Driven Formal Methods for Cyber-Physical Systems control

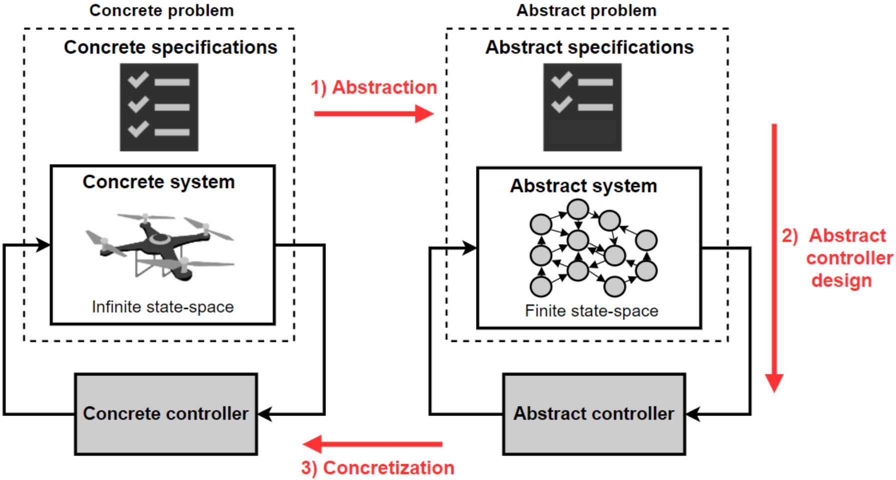
European Research Council

Established by the European Commission

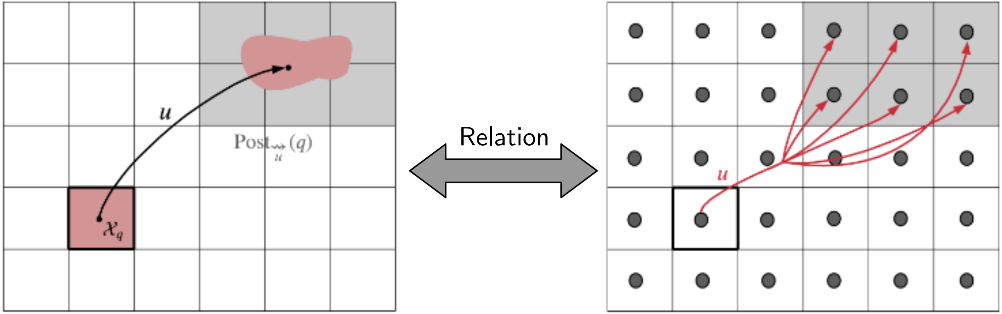
- Generic but modular
- Opportunistic
- State-space driven
- Safety-critical
- Scalable
- Logic-enhanced
- Data-friendly
- ...

Bottom-up approach

Abstraction-based control



Classical abstraction-based control



Additional non-determinism resulting from the discretization

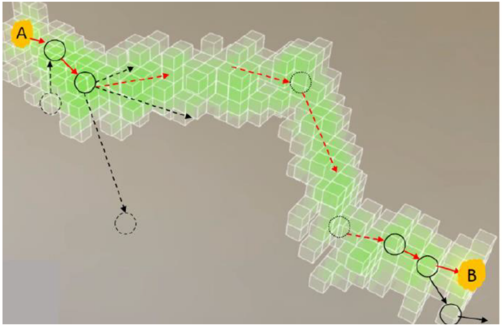
The number of cells grows exponentially with the dimension of the state space

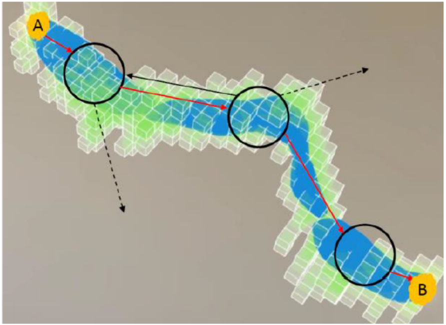
Curse of dimensionality

Smart abstraction

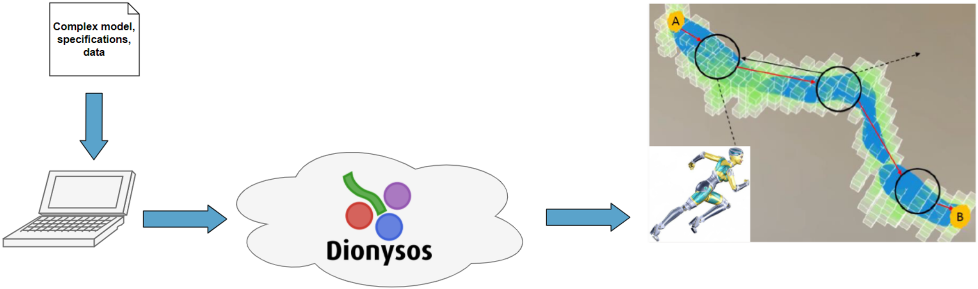
Co-design the abstraction and the controller

⇒ Partially discretize the state space with non-uniform cells with respect to the specific control task.





Dionysos



The objectives of Dionysos are as follows

- Implement our state-of-the-art smart abstraction algorithms developed in L2C.
- Implement existing algorithms in our modular framework and demonstrate the effectiveness of the Julia language.

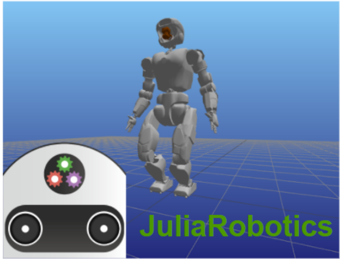
Main contributors: Benoît Legat, Julien Calbert, Adrien Banse, Lucas N. Egidio

Dionysos in Julia

JuliaReach

JuMP

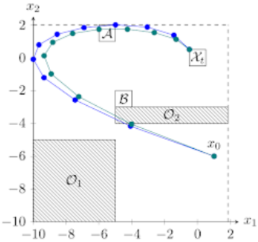
julia

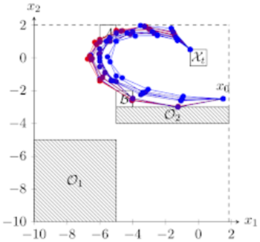


Dionysos

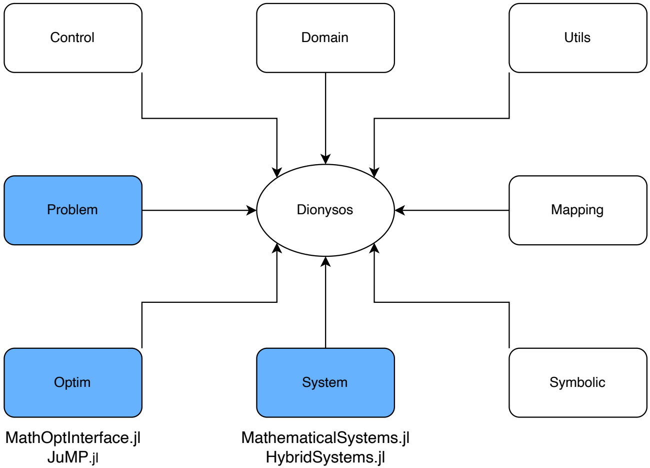
Hybrid optimal control solver combining:

- Smart abstraction
- Q-learning
- Branch and bound
- Path-Complete
- Sum-of-Squares
- ...



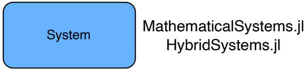


Package structure



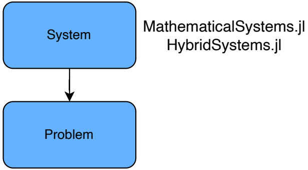
Package structure: System

- Structures for mathematical definitions of
  - Control dynamical systems $x^+ = f(x, u)$
  - Controllers $u(x) = K(x)$
- Methods
  - For example: Runge Kutta scheme
- Built on top of
  - JuliaReach/MathematicalSystems.jl
  - blegat/HybridSystems.jl



Package structure: Problem

- Structures to define control problems
- For now, two types of problem
  - `OptimalControlProblem`: initial, target and cost
  - `SafetyProblem`: safe/unsafe sets
- Each problem is composed of a system, and problem specifications



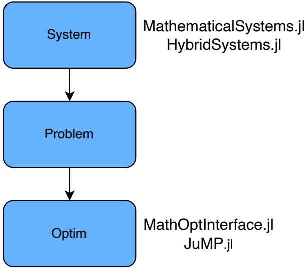
Package structure: Optim

- Methods to solve the problems, the optimizers
- `src/optim`

```
- abstraction
    - SCOTS_abstraction.jl
    - ellipsoids_abstraction.jl
    - hierarchical_abstraction.jl
    - lazy_abstraction.jl
    - lazy_ellipsoids_abstraction.jl
- bemporad_morari.jl
- branch_and_bound.jl
- q_learning.jl
```

- Built on top of
  - jump-dev/MathOptInterface.jl
  - jump-dev/JuMP.jl

  Every optimizer is a subtype of `MOI.AbstractOptimizer`
- Each optimizer is composed of a problem, and method specifications



Examples

Now, let's focus on two examples

1. Implementation of a smart abstraction method on a simple problem
2. Implementation of a abstraction method from state-of-the-art, and comparison to existing implementations

First example: Simple problem

Consider the very simple discrete-time system

$$x_{t+1} = x_t + hu,$$

where $h \in \mathbb{R}$ is a time step, $x_t \in \mathbb{R}^2$ is the state and $u \in \mathbb{R}^2$ is an input.

Control objective = Drive the state x from an initial position to a target position while avoiding obstacles

For that, we will use a smart abstraction method presented in [Calbert et al., 2021], called hierarchical abstractions.

[Calbert et al., 2021] Julien Calbert, Benoit Legat, Lucas N. Egidio and Raphaël M. Jungers. 2021. In Proceedings of the 60th IEEE Conference on Decision and Control (CDC).

First example: Simple problem (sketch of the code)

- First, we define the system

```
function system(
    rectX,
    obstacles,
    rectU,
    Uobstacles,
    tstep,
    measnoise,
    periodic,
    periods,
    T0,
)
    ...
    return SimpleSystem(...)
sys = system(...)
```

First example: Simple problem (sketch of the code)

- We then define the problem

```
problem = OptimalControlProblem(
    sys,
    initial_set,
    target_set,
    state_cost,
    transition_cost,
    N
)
```

First example: Simple problem (sketch of the code)

- And finally we can define the smart abstraction method (an optimizer)

```
const AB = Dionysos.Optim.Abstraction
optimizer = MOI.instantiate(AB.HierarchicalAbstraction.Optimizer)
AB.HierarchicalAbstraction.set_optimizer!(
    optimizer,
    concrete_problem,
    hx_global,
    Ugrid,
    compute_reachable_set,
    minimum_transition_cost,
    local_optimizer,
    max_iter,
    max_time;
    option = true,
)
```

First example: Simple problem (sketch of the code)

- We solve the whole problem

```
MOI.optimize!(optimizer)
```

- And we can extract, for example, its abstract system

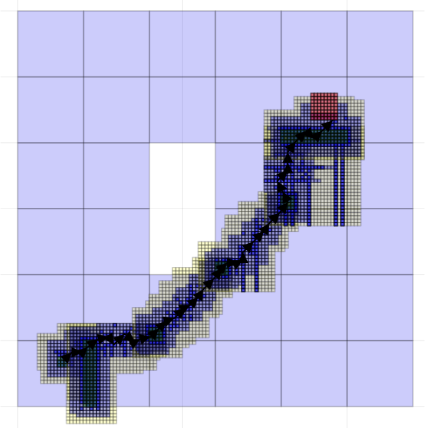
```
abstract_system = MOI.get(optimizer, MOI.RawOptimizerAttribute("abstract_system"))
```

- Go to Documentation > Examples > Hierarchical-abstraction for the full example!

First example: Simple problem (sketch of the code)

```
fig = plot(; aspect_ratio = :equal);

plot!(
    optimizer.hierarchical_problem;
    path = optimizer.optimizer_BB.best_sol,
    heuristic = false,
    fine = true,
)
plot!(UT.DrawTrajectory(x_traj); ms = 0.5)
```



Second example: Path planning

Consider the model of a vehicle in the 2-dimensional plane given by

$$\dot{x} = f(x,u) = \begin{pmatrix} u_1 \cos(\alpha + x_3) \cos(\alpha^{-1}) \\ u_1 \sin(\alpha + x_3) \cos(\alpha^{-1}) \\ u_1 \tan(u_2) \end{pmatrix},$$

with $U = [-1,1] \times [-1,1]$, and $\alpha = \arctan(\tan(u_2)/2)$.

- (x_1, x_2) is the position,
- x_3 is the orientation of the vehicle,
- u_1 is the rear wheel velocity,
- u_2 is the steering angle.

We study the sampled problem with a sampling time τ.

Second example: Path planning

- Control objective = Drive the vehicle from an initial position to a target position while avoiding obstacles.
- To solve this problem, we use our implementation of an abstraction method described in [Reissig et al., 2017]
- Let's have an overview of how it looks like using Dionysos.jl...

[Reissig et al., 2017] G. Reissig, A. Weber and M. Rungger. 2017. Feedback Refinement Relations for the Synthesis of Symbolic Controllers. In IEEE Transactions on Automatic Control, vol. 62, no. 4, pp. 1781-1796.

Second example: Path planning (sketch of the code)

- We first choose the right optimizer

```
optimizer = MOI.instantiate(AB.SCOTSAbstraction.Optimizer)
```

- We then set the concrete problem to the optimizer

```
MOI.set(
    optimizer,
    MOI.RawOptimizerAttribute("concrete_problem"),
    concrete_problem
)
```

Where `concrete_problem` is defined in `problems/path_planning.jl`.

Second example: Path planning (sketch of the code)

- Now, we define the state/input grids

```
MOI.set(optimizer, MOI.RawOptimizerAttribute("state_grid"), state_grid)
MOI.set(optimizer, MOI.RawOptimizerAttribute("input_grid"), input_grid)
```

- We solve the problem

```
MOI.optimize!(optimizer)
```

Second example: Path planning (sketch of the code)

- Our solver then creates an abstract problem, finds an abstract controller, and refines it to a concrete controller

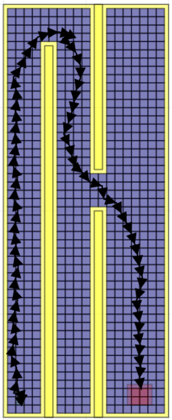
```
abstract_system = MOI.get(
    optimizer, MOI.RawOptimizerAttribute("abstract_system")
)
abstract_problem = MOI.get(
    optimizer, MOI.RawOptimizerAttribute("abstract_problem")
)
abstract_controller = MOI.get(
    optimizer, MOI.RawOptimizerAttribute("abstract_controller")
)
concrete_controller = MOI.get(
    optimizer, MOI.RawOptimizerAttribute("concrete_controller")
)
```

Second example: Path planning (sketch of the code)

- Let's now extract the closed-loop trajectory and plot the result

```
x_traj, u_traj = ...
# ... Plotting the domain thanks
# ... to implemented Recipes
plot!(UT.DrawTrajectory(x_traj); ms = 0.5)
```

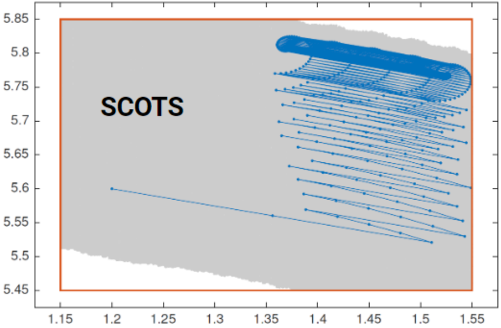
- Go to Documentation > Examples > Path Planning for the full example!

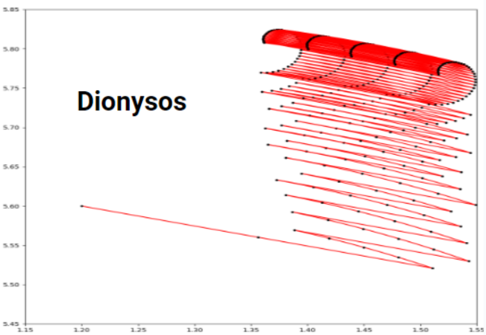


Preliminary benchmarks

Planar switched affine system with univariate control and 2 modes

	Abstraction [s]	Synthesis [s]
Pessoa	478.7	65.2
SCOTS	18.1	75.4
Dionysos	1.02	0.22

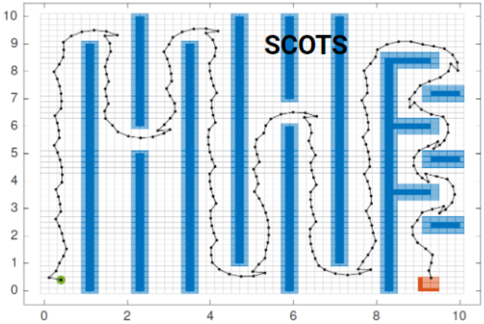


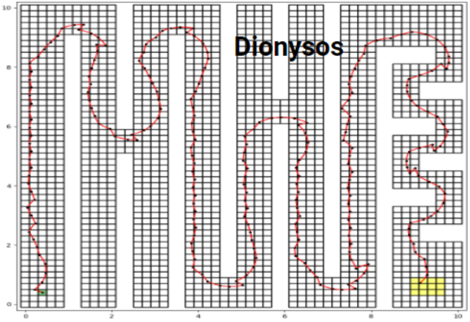


Preliminary benchmarks

Nonlinear system with 3 states, 2 inputs, obstacles and target

	Abstraction $[s]$	Synthesis $[s]$
Pessoa	13509	535
SCOTS	53	210
Dionysos	6.59	0.57





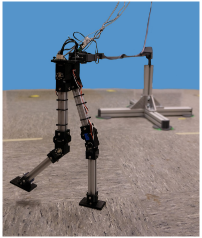
Conclusions

In summary

- Dionysos implements state-of-the-art and smart abstraction methods to solve control problems for complex dynamical systems
- It offers a common framework thanks to its implementation based on JuMP and MathOptInterface
- It is highly modular and benefits from the power/convenience of many other Julia packages

Future work

- Solving the 27 issues on the github...
- Implementation of an orchestrator
- Benchmarking Dionysos on our walking robot



Thank you for listening!

Dionysos UCLouvain icteam

European Research Council
Established by the European Commission

julia

https://github.com/dionysos-dev/Dionysos.jl

References

About Dionysos

[1] B. Legat, R. M. Jungers, and J. Bouchat, Abstraction-based branch and bound approach to Q-learning for hybrid optimal control, in Proceedings of the 3rd Conference on Learning for Dynamics and Control, 2021, pp. 263–274.

[2] J. Calbert, B. Legat, L. N. Egidio, and R. Jungers, Alternating Simulation on Hierarchical Abstractions, 2021 60th IEEE Conference on Decision and Control (CDC), 2021.

[3] L. N. Egidio, T. A. Lima, and R. M. Jungers, State-feedback Abstractions for Optimal Control of Piecewise-affine Systems, 2022 IEEE 61st Conference on Decision and Control (CDC), 2022.

About other toolboxes

[1] M. Mazo, A. Davitian, and P. Tabuada, PESSOA: A Tool for Embedded Controller Synthesis, Computer Aided Verification, pp. 566–569, 2010.

[2] M. Rungger and M. Zamani, SCOTS, Proceedings of the 19th International Conference on Hybrid Systems: Computation and Control, 2016.

[3] S. Mouelhi, A. Girard, and G. Gössler, CoSyMA, Proceedings of the 16th international conference on Hybrid systems: computation and control, 2013.